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Structural Redesign for Forced Response with Proportional Damping by Large Admissible Perturbations

Michael M. Bernitsas* and Vincent Y. Blouin†
University of Michigan, Ann Arbor, Michigan 48109-2145

A method to solve redesign (inverse design) problems of complex structures with forced response amplitude constraints is developed. The assumption is made that a structure is excited by harmonic external forces at a given frequency. The problem is to find optimum values of structural characteristics in order to achieve a desired level of forced response at one or several locations on the structure. The method of large admissible perturbations (LEAP) is used. The main advantage of LEAP is that a solution of the inverse problem can be found automatically without trial and error or repetitive finite element analyses. Using this method, algorithms with modal dynamic constraints and static displacement constraints have been developed and tested in the past. In this paper the redesign problem with forced response amplitude constraints is formulated. The method consists of two distinctive parts. First, the general perturbation equations are derived. They provide relations between the original structure S1, which is known and has undesired forced response amplitudes, and the unknown objective structure S2. Second, the redesign problem is solved by an incremental prediction–correction scheme, which permits large changes in redesign. Structural damping is considered in the form of Rayleigh damping. Under this formulation the damping matrix can be diagonalized by use of the real mode shapes of the undamped structure, which leads to the derivation of an exact perturbation equation with no loss of accuracy. Modal dynamic and static constraints may also be imposed simultaneously by the designer. The algorithm produces accurate results for large changes in response amplitude without additional finite element analyses. The number of extracted modes and the increment size control the accuracy of the results and the computational time. The importance of the choice of the objective function is discussed, and examples are presented.

Nomenclature

$[A]$ = admixture matrix
A_{ij} = admixture coefficient, participation of jth mode to changes in ith mode
a_1, a_2 = damping coefficients in proportional damping model
$[c]$ = damping matrix
$\{d\}$ = forced response amplitude vector
e = eth structural redesign variable ($e = 1, \ldots, p$)
$\{g'\}$ = transformed displacement vector
$[{}^\backslash K_\backslash], [{}^\backslash M_\backslash]$ = generalized stiffness and mass matrices of baseline structure
$[k], [m]$ = stiffness and mass matrices of baseline structure
$[k_e], [m_e]$ = invariant parts of stiffness and mass matrices for the eth redesign variable
n_d = number of constraints in optimization problem
n_{dof} = number of degrees of freedom
n_r = number of extracted modes
p = number of structural redesign variables
S1 = initial structure, initial finite element model
S2 = desired structure, updated finite element model
α_e = fractional changes (redesign variables)
γ = penalty coefficient in objective function
Δ = prefix denoting large changes
$[\Delta k]$ = large change to stiffness matrix
$[\Delta m]$ = large change to mass matrix
$[\Delta\Phi]$ = matrix of mode shape vector changes relative to baseline structure
$\Delta\omega_i$ = large change to ith baseline natural frequency, rad/s
$[\Phi]$ = matrix of mode shape vectors of baseline structure
$\{\phi\}_i$ = ith mode shape vector
ω_i = ith natural frequency of baseline structure, rad/s
$[{}^\backslash{}_\backslash]$ = diagonal matrix

Subscripts and Superscripts

$[\,]^T, \{\,\}^T$ = transpose of a matrix and a vector, respectively
$(\,)'$ = objective structure
${}_l(\,)$ = preindex referring to the lth increment

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*Professor, Department of Naval Architecture and Marine Engineering, 2600 Draper Road.

†Research Assistant, Department of Naval Architecture and Marine Engineering, 2600 Draper Road.

I. Introduction

STRUCTURES, such as cars, airplanes, ships, offshore platforms, etc., are subject to various dynamic excitations, which induce structural vibrations. Excessive levels of vibration may result in structural damage, equipment malfunction, machinery failure, or crew discomfort.[1,2] Descriptions of the sources of excitation can be found in the literature.[1,3,4] Some of the most common sources of excitation are the main propulsion engine, secondary engines, and propellers. In most cases vibrations caused by the excitation provided by the propulsion system are the most serious and must be addressed in the early design stages.[2,4]

For a given dynamic harmonic excitation a redesign objective is to reduce local vibrations in the structure to acceptable limits. This is an inverse design problem, also known as a synthesis problem, which is to find the changes within the structure that lead to a prescribed response. The problem is also called structural redesign, which corresponds to a certain class of optimization problems. Designers can define several redesign variables such as cross-sectional areas, moments of inertia of beams, and plate thicknesses. Then, the goal is to find the values of the redesign variables that produce an improved structure in the sense of reduced vibration amplitudes.

Structural redesign can require numerous finite element analyses (FEA), making the design spiral process long, expensive, and possibly inconclusive. It can even be unsuccessful in cases of multiple requirements. Sensitivity methods also require several FEA and are limited to small structural changes. A multitude of approaches based on design sensitivity analysis can be found in the literature. An overview of the research conducted on structural optimum design by sensitivity analysis (DSA) is provided by Kajiwara and Nagamatsu.[5] Hsieh and Arora[6] described and compared two ways of calculating design sensitivity coefficients for the optimization of dynamic response of structural systems. Watts and Starkey[7] proposed an optimization technique based on DSA to improve the linear structural response of viscously damped structures. Abrahamsson[8] used DSA to reduce the peak magnitude at a specific frequency of excitation and the average response magnitude over a frequency range. Lin et al.[9] developed an algorithm for dynamic response optimization based on design sensitivity analysis of nonlinear structural systems including the effects of viscous damping and design-dependent loads. Nonlinear vibrations and response-dependent loading were introduced also into the problem by Chen et al.[10] Ting et al.[11] used DSA for the refinement of finite element model in order to achieve acceptable correlation between the predicted and the actual forced responses of structures. Inoue et al.[12] proposed an optimization method to reduce the vibration of thin-plate structures. The technique of modal analysis was used to derive the sensitivity of the vibration energy with respect to the design variables. Bucher and Braun[13,14] dealt with the problem of reducing the vibratory response created by random narrowband and deterministic transient excitations. The response, the cost function, and the sensitivity coefficients were calculated by modal decomposition. Ma et al.[15] employed two sensitivity analysis methods, the direct frequency response method and the modal frequency response method, to calculate the sensitivity of the objective function for the structural topology and shape optimization for frequency response problems.

Matsumoto et al.[16] proposed an approach based on a weighted minimax method to solve a design optimization problem aiming at reducing maximum vibration levels. They considered excitations in multiple directions and for a wide frequency band.

Bouazzouni et al.[17] used an approximate Ritz technique to achieve the reanalysis of the frequency response functions of modified structures. They showed that for a better representation of the frequency response the original Ritz basis should be extended with a set of additional vectors resulting from the static contribution of the neglected eigenvectors.

Most of the methods used today are based on design sensitivities and require a large number of finite element analyses, which can become extremely time consuming in the case of large structures. Alternatively, the method of large admissible perturbations (LEAP) is a general methodology, which solves redesign (inverse design) problems of complex structures without trial and error or repetitive finite element analyses. This technique consists of two parts: the perturbation approach to redesign (PAR), which formulates the redesign problem as a two-state problem; and the LEAP algorithm, which calculates the objective structure from the results of the FEA of the initial design and the designer's specifications. This approach to redesign has been used successfully for large or small changes in redesign with static deformation constraints and/or modal dynamic constraints. The methodology and the LEAP algorithms are implemented in RESTRUCT, a 30,000-Fortran-command code.[18] The large admissible perturbations theory was developed to solve several inverse problems such as structural redesign for free vibrations,[19–25] static response,[23–26] stress levels,[27] redundancy and reliability of marine structures,[28,29] resizing for model correlation,[30,31] and topology redesign.[32] With LEAP redesign for large changes on the order of 100–200% has been achieved with satisfactory accuracy and no repeated FEA.

In this paper accuracy is defined as the difference between the desired response specified by the designer and the response computed by the FEA of the new structure produced by redesign.

II. Theory of Structural Redesign for Forced Response Amplitude

The problem of structural redesign is defined as follows. A structure or a structural model has a response, which is unacceptable. For example, an offshore oil production platform may have a natural frequency in the range of the wave excitation, or a machine shaft may have excessive displacements for alignment, or the vibrations of a deck to machinery excitation may be too high. A modified structure or model must be produced that will have acceptable response according to the designer's specifications. The designer must specify the structural response that should be changed and the structural properties that are allowed to change; the latter would be the redesign variables. In this paper a methodology based on large admissible perturbations is developed to solve the problem of structural redesign (inverse design) for forced response amplitude.

A. Perturbation Approach to Redesign

Two finite element models are considered: the initial finite element model S1, also referred to as the baseline model with undesired response; and the desired finite element model S2, also referred to as the objective structure. The structural redesign problem is formulated as a constrained optimization problem, which is presented in the following section. The general perturbation equations relating response of the baseline to the objective structure are derived next. These provide constraints in the redesign optimization problem. The process of relating the two structures is called PAR.

First, the perturbation equations for the undamped case are derived. Then, structural damping is introduced in the form of proportional damping. Finally, the computation of the unknown eigenvectors by linear predictions is presented.

Undamped Structure

For the initial structure S1 the equation for the forced response amplitude in the frequency domain is, in matrix form:

$$\big([k] - \omega_0^2[m]\big)\{d\} = \{f\} \tag{1}$$

where $[k]$ and $[m]$ are the stiffness and mass matrices, $\{d\}$ is the response amplitude vector, and ω_0 and $\{f\}$ are the frequency and the amplitude of excitation on the structure.

In theory this equation is complex. If the phase reference is set equal to that of the excitation, i.e., zero complex argument for $\{f\}$, then the response $\{d\}$ is in phase with the excitation because of the lack of damping. Then, Eq. (1) and the following equations are real.

All quantities related to S2 are primed. In this paper we assume that the frequency and the amplitude of the excitation are the same for the initial and the objective structures. This is a legitimate assumption because typical sources of excitations such as engines, compressors, etc., induce excitation, which is independent of the structure and its response. That is, it is assumed that $\omega_0' = \omega_0$, $\{f'\} = \{f\}$. Thus,

$$\big([k'] - \omega_0^2[m']\big)\{d'\} = \{f\} \tag{2}$$

S1 and S2 quantities can be related by the following equations:

$$[k'] = [k] + [\Delta k] \tag{3}$$

$$[m'] = [m] + [\Delta m] \tag{4}$$

$$\{d'\} = \{d\} + \{\Delta d\} \tag{5}$$

where the prefix Δ indicates total difference between an S1 quantity and its S2 counterpart.

Let us introduce the transformed displacement vector $\{g'\}$ as follows:

$$\{d'\} = \{d\} + [\Phi']\{g'\} \tag{6}$$

where $[\Phi']$ is the matrix of mode shape vectors of the objective structure,

$$[\Phi'] = \big[\{\phi'\}_1, \ldots, \{\phi'\}_{n_r}\big] \tag{7}$$

In Eq. (6) the vector $\{d\}$ is the equivalent of what is called in other types of applications static mode compensation. This additional vector was first introduced in the redesign procedure by Bernitsas and Suryatama[8] for the development of the static perturbation equation. The arbitrary number of mode shapes, called extracted modes, used in matrix $[\Phi']$ has an impact on precision and computational time. This issue will be discussed later.

Substituting Eq. (6) into Eq. (2) gives

$$\big([k'] - \omega_0^2[m']\big)\{d\} + \big([k'] - \omega_0^2[m']\big)[\Phi']\{g'\} = \{f\} \tag{8}$$

Premultiplying Eq. (8) by $[\Phi']^T$ and introducing the generalized stiffness and mass matrices $[^\backslash K'_\backslash]$ and $[^\backslash M'_\backslash]$, where the notation $[^\backslash{}_\backslash]$ indicates a diagonal matrix, yields

$$[\Phi']^T([k'] - \omega_0^2[m'])\{d\} + ([^\backslash K'_\backslash] - \omega_0^2[^\backslash M'_\backslash])\{g'\} = [\Phi']^T\{f\} \quad (9)$$

$$[^\backslash K'_\backslash] = [\Phi']^T[k'][\Phi'] \quad (10)$$

$$[^\backslash M'_\backslash] = [\Phi']^T[m'][\Phi'] \quad (11)$$

Substituting Eqs. (3) and (4) into Eq. (9) and eliminating terms using Eq. (1) gives

$$[\Phi']^T([\Delta k] - \omega_0^2[\Delta m])\{d\} + ([^\backslash K'_\backslash] - \omega_0^2[^\backslash M'_\backslash])\{g'\} = \{0\} \quad (12)$$

The main advantage of the generalized matrices is their diagonal form, which allows extraction of the unknown $\{g'\}$ without numerical inversion of matrices. Then,

$$\{g'\} = -([^\backslash K'_\backslash]^{-1} - \omega_0^2[^\backslash M'_\backslash]^{-1})[\Phi']^T([\Delta k] - \omega_0^2[\Delta m])\{d\} \quad (13)$$

Let us emphasize this property by noting that the inverse matrices $[^\backslash K'_\backslash]^{-1}$ and $[^\backslash M'_\backslash]^{-1}$ are also diagonal and that each term can be written as

$$([^\backslash K'_\backslash]^{-1})_{ii} = 1/([^\backslash K'_\backslash])_{ii} = 1/(\{\phi'\}_i^T[k]\{\phi'\}_i + \{\phi'\}_i^T[\Delta k]\{\phi'\}_i) \qquad i = 1, \ldots, n_{\text{dof}} \quad (14)$$

$$([^\backslash M'_\backslash]^{-1})_{ii} = 1/([^\backslash M'_\backslash])_{ii} = 1/(\{\phi'\}_i^T[m]\{\phi'\}_i + \{\phi'\}_i^T[\Delta m]\{\phi'\}_i) \qquad i = 1, \ldots, n_{\text{dof}} \quad (15)$$

Finally, substituting $\{g'\}$ from Eq. (13) into Eq. (6) produces the general perturbation equation for the forced response amplitude:

$$\{d'\} = \{d\} - [\Phi']([^\backslash K'_\backslash]^{-1} - \omega_0^2[^\backslash M'_\backslash]^{-1})[\Phi']^T([\Delta k] - \omega_0^2[\Delta m])\{d\} \quad (16)$$

In this vectorial equation, the matrices $[\Delta k]$ and $[\Delta m]$ are functions of the unknowns (redesign variables). In the present study the shape of the structure is unchanged. Properties such as thicknesses of plates and pipes, cross-sectional areas, or moments of inertia can be chosen as the redesign variables. The finite element model is divided into groups of elements, called element sets, such that all of the elements within a group have the same redesign variables. Let us define α_e as the fractional change of a property in element set e. For instance, if A and A' are the cross-sectional areas of the eth element set in S1 and S2, respectively, then α_e is such that

$$A'/A = (1 + \alpha_e) \quad (17)$$

Consequently, changes in stiffness and mass matrices with linear dependency can be expressed by

$$[\Delta k] = \sum_{e=1}^{p}[\Delta k_e] = \sum_{e=1}^{p}[k_e]\alpha_e \quad (18)$$

$$[\Delta m] = \sum_{e=1}^{p}[\Delta m_e] = \sum_{e=1}^{p}[m_e]\alpha_e \quad (19)$$

Equations (18) and (19) correspond to rod and beam elements. In the case of plates, the stiffness matrix dependency on α_e is nonlinear (cubic function in α_e) as derived in Ref. 24. As a result, the perturbation equations are more complicated, but the concept of formulating and solving the redesign problem by large admissible perturbations is still valid.

The general perturbation equation (16) is a vectorial equation from which one or several components can be considered as constraints. For the ith component we have

$$d'_i = d_i - \sum_{m=1}^{n_r} \frac{\phi'_{i,m}\sum_{e=1}^{p}(\{\phi'\}_m^T[z_e]\{d\})\alpha_e}{\{\phi'\}_m^T[z]\{\phi'\}_m + \sum_{e=1}^{p}(\{\phi'\}_m^T[z_e]\{\phi'\}_m)\alpha_e} \quad (20)$$

where d_i, d'_i are the amplitudes of the forced vibration of the ith degree of freedom for the initial and the objective structures, respectively, d_i is computed by the initial finite element analysis and d'_i is prescribed by the designer; $[z] = [k] - \omega_0^2[m]$; $[z_e] = [k_e] - \omega_0^2[m_e]$; $\{\phi'\}_m$ is the mth eigenvector of the desired structure; $\phi'_{i,m}$ is the ith component of the mth eigenvector of the desired structure; n_r is the number of extracted modes used in the modal decomposition; and α_e is the fractional changes in structural properties.

The general perturbation equation presented here, Eq. (20), corresponds to the case of bar elements where the dependency of the structural matrices on the fractional changes is linear. In the case of plate elements, the general perturbation equation is similar but with a nonlinear dependency.[24]

Proportionally Damped Structure

For a realistic structural model of a physical problem, several sources of damping with different mathematical models should be considered. As described earlier, the advantage of the generalized stiffness and mass matrices is their diagonal form, which allows a simple inversion of matrices in the development of the general perturbation equations. By introducing a damping matrix in the finite element formulation, diagonalization of the generalized damping matrix is not possible. Most important, the explicit dependency on α_e is lost.[22] The only case in which the diagonal form is maintained is that of proportional damping (Rayleigh damping), where the damping matrix is expressed as a linear function of the stiffness and mass matrices.

In the frequency domain the equation of equilibrium with damping is, in matrix form,

$$([k] + i\omega_0[c] - \omega_0^2[m])\{d\} = \{f\} \quad (21)$$

This equation is in complex form and therefore the displacement vector contains real and imaginary parts. In the case of proportional damping, the matrix damping $[c]$ can be expressed as

$$[c] = a_1[k] + a_2[m] \quad (22)$$

where a_1 and a_2 are two constants.

Then, following the same derivation as in the undamped case, the general perturbation equation for the forced response amplitude with proportional damping, is, in scalar form,

$$d'_i = d_i - \sum_{m=1}^{n_r} \frac{\phi'_{i,m}\sum_{e=1}^{p}(\{\phi'\}_m^T[z_e]\{d\})\alpha_e}{\{\phi'\}_m^T[z]\{\phi'\}_m + \sum_{e=1}^{p}(\{\phi'\}_m^T[z_e]\{\phi'\}_m)\alpha_e} \quad (23)$$

with

$$[z] = (1 + ia_1\omega_0)[k] - (\omega_0^2 - ia_2\omega_0)[m]$$

$$[z_e] = (1 + ia_1\omega_0)[k_e] - (\omega_0^2 - ia_2\omega_0)[m_e]$$

As opposed to the undamped case, the general perturbation equation is expressed in complex form, which cannot be handled by the optimizer[33] used in code RESTRUCT. Therefore, the complex equation is divided into real and imaginary parts, or amplitude and argument, to produce a pair of real general perturbation equations.

Linear Predictions of Eigenvectors

In Eqs. (20) and (23) the unknowns are the fractional changes $\alpha_e, e = 1, \ldots, p$. As explained in Sec II.B, however, the eigenvectors $\{\phi'\}_m$, $m = 1, \ldots, n_r$ are unknown. The small perturbation method developed by Stetson et al.[19] and Sandström et al.[20] gives perturbation equations that relate the unknown eigenvectors of the desired structure to the known eigenvectors of the initial structure. Their derivation is briefly repeated in this section for completeness of the presentation.

In matrix form the ith mode free vibration equation for the initial and desired structures can be written as

$$[k]\{\phi\}_i = [m]\{\phi\}_i\omega_i^2 \quad (24)$$

$$[k']\{\phi'\}_i = [m']\{\phi'\}_i\omega_i'^2 \quad (25)$$

The perturbation relations are

$$\{\phi'\}_i = \{\phi\}_i + \{\Delta\phi\}_i \tag{26}$$

$$\omega_i'^2 = \omega_i^2 + \Delta\omega_i^2 \tag{27}$$

Premultiplying Eq. (25) by $\{\phi\}_i^T$ and using Eqs. (3), (4), (25), and (26), we derive an equation that can be developed into 24 terms among which 15 are nonlinear terms in $[\Delta()]^2$ and $[\Delta()]^3$. From this point we assume small perturbations to linearize this equation as

$$\begin{aligned}\{\phi\}_j^T[\Delta k]\{\phi\}_i - \{\phi\}_j^T[\Delta m]\{\phi\}_i\omega_i^2 &= \{\phi\}_j^T[k]\{\Delta\phi\}_i \\ &- \{\Delta\phi\}_j^T[k]\{\phi\}_i + \{\phi\}_j^T[m]\{\Delta\phi\}_i\omega_i^2 \\ &+ \{\Delta\phi\}_j^T[m]\{\phi\}_i\omega_i^2 + \{\phi\}_j^T[m]\{\phi\}_i\Delta\omega_i^2\end{aligned} \tag{28}$$

For $i = j$ using Eq. (24) and its transpose yields

$$\{\phi\}_i^T[\Delta k]\{\phi\}_i - \{\phi\}_i^T[\Delta m]\{\phi\}_i\omega_i^2 = M_i\Delta\omega_i^2 \tag{29}$$

where M_i is the ith diagonal term of the generalized mass matrix. For $i \neq j$, $\{\phi\}_j^T[m]\{\phi\}_i = 0$ and using the transpose of Eq. (24) for the jth mode into Eq. (28) yields

$$\{\phi\}_j^T[\Delta k]\{\phi\}_i - \{\phi\}_j^T[\Delta m]\{\phi\}_i\omega_i^2 = \{\phi\}_j^T[m]\{\Delta\phi\}_i\left(\omega_i^2 - \omega_j^2\right) \tag{30}$$

Let us assume that the changes in eigenvectors can be written as

$$[\Delta\Phi] = [\Phi][A]^T \tag{31}$$

where $[A]$ is the matrix of admixture coefficients with $A_{ii} = 0$ and A_{ij} small for i different from j. One can see that $\{\phi\}_j^T[m] \times \{\Delta\phi\}_i = M_j A_{ij}$, which gives

$$\begin{aligned}A_{ij} = \frac{1}{M_j\left(\omega_i^2 - \omega_j^2\right)} \sum_{e=1}^{p} \Big(&\{\phi'\}_j^T[\Delta k_e]\{\phi'\}_i \\ &- \{\phi'\}_j^T[\Delta m_e]\{\phi'\}_i\omega_i^2\Big)\alpha_e\end{aligned} \tag{32}$$

where M_j is the jth diagonal term of the generalized mass matrix.

Equations (26), (31), and (32) provide the relation between the modes of the initial structure and those of the new structure where we assumed small changes in the derivation of Eq. (32). For large changes, a correction step along with an incremental approach must be introduced, as described next.

B. LEAP Algorithm

Formulation as an Optimization Problem

The redesign problem can be formulated as an optimization problem. The objective function may be any function of the design variables (fractional changes). In this study we consider the minimum change criterion, which is to find the closest structure to the initial design. Then, the optimization problem with forced response amplitude constraints can be formulated as follows.

Minimize:

$$\sum_{e=1}^{p} \alpha_e^2 \tag{33}$$

subject to n_d forced response amplitude requirements, $i = 1, \ldots, n_d$:

$$d_i' = d_i - \sum_{m=1}^{n_r} \frac{\phi_{i,m}' \sum_{e=1}^{p}\left(\{\phi'\}_m^T[z_e]\{d\}\right)\alpha_e}{\{\phi'\}_m^T[z]\{\phi'\}_m + \sum_{e=1}^{p}\left(\{\phi'\}_m^T[z_e]\{\phi'\}_m\right)\alpha_e} \tag{34}$$

and $2p$ lower and upper bounds on redesign variables, $e = 1, \ldots, p$:

$$-1 < \alpha_e^- \leq \alpha_e \leq \alpha_e^+ \tag{35}$$

where α_e^- and α_e^+ are pairs of bounds set by the designer for each design variable.

Objective Function

In the optimization problem Eq. (33) corresponds to a minimum change criterion. In this case the solution found by the optimizer corresponds to the structure that satisfies the constraints and is the closest to the initial design. For extreme cases of large changes, this objective function leads sometimes to solutions with poor accuracy because minimizing the total changes tends to concentrate the changes in one portion of the structure rather than distributing the changes uniformly. A nonuniform change on the structure results is a rather large change in mode shapes. On the contrary, a uniform change on the entire structure has little effect on the mode shapes. Consequently, in the latter case, the term $\{\Delta\phi\}_i$ in Eq. (26) is even more negligible, and the linearization of Eq. (25) is even more justified for better accuracy. Following this idea, an alternative objective function is proposed, which contains a penalty term, that minimizes the difference between redesign variables of adjacent element sets, as follows.

Minimize:

$$\left\{\sum_{e=1}^{p}\alpha_e^2 + \gamma\sum_{e=1}^{p}(\alpha_e - \alpha_{e+1})^2\right\} \tag{36}$$

where γ is an arbitrary penalty coefficient. This objective function is tried for the first time in this paper. It provides satisfactory results presented in the following sections.

In a large class of engineering problems, the objective is to minimize the weight of the structure. In this case a uniform distribution of changes is not guaranteed.

As far as feasibility is concerned, if the problem is underdetermined, i.e., when the number of constraints is such that the solution is not unique, then the acceptable solution is the one that satisfies the optimization criterion. In the case of an overdetermined problem, the solution may not exist, and a minimum error algorithm is used by the optimizer.[22]

Incremental Procedure

For large total changes the process involves several sources of nonlinearities. If the optimization problem was solved directly as presented in Eqs. (33–35), a numerical solution could be found. Because of the presence of nonlinearities, however, there is no guarantee that this solution would be physically correct. This concept is schematically represented in Fig. 1.

To overcome this difficulty, an incremental procedure is used with the introduction of a nonlinear correction step following the linear prediction in each increment. The total large desired changes in forced response is divided into N small incremental changes. From this point all of the variables can be rewritten with the pre-index l, which refers to the lth increment.

Prediction–Correction Scheme at Each Increment

In each increment the optimization problem formulated by Eqs. (33–35) has to be solved for the unknown fractional changes, ${}_l\alpha_e$, $e = 1, \ldots, p$ (Ref. 33). The eigenvectors ${}_l\{\phi'\}_m$, $m = 1, \ldots, n_r$, in Eq. (34), however, are unknown. This is handled by the prediction–correction scheme at each increment. The idea is to

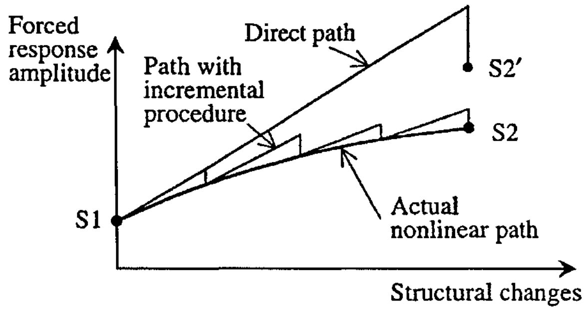

Fig. 1 Schematic representation of solution process: S1, initial structure; S2, redesigned structure; and S2′, inaccurately redesigned structure (due to excessive change in one increment, direct path).

approximate the eigenvectors ${}_l\{\phi'\}_m$ with the eigenvectors of the previous increment ${}_{(l-1)}\{\phi'\}_m$ in the prediction phase. Then, the optimization problem, Eqs. (33–35), can be solved for the fractional changes ${}_l\alpha_e$, $e = 1, \ldots, p$, which provides a prediction of the solution. From this predicted solution the eigenvectors ${}_l\{\phi'\}_m$ can be computed using Eqs. (26), (31), and (32). In the correction phase the optimization problem, Eqs. (33–35), is solved with the new eigenvectors, ${}_l\{\phi'\}_m$, to give the corrected solution for the fractional changes ${}_l\alpha_e$, $e = 1, \ldots, p$.

LEAP Algorithm

A schematic representation of the algorithm is given in Fig. 2.

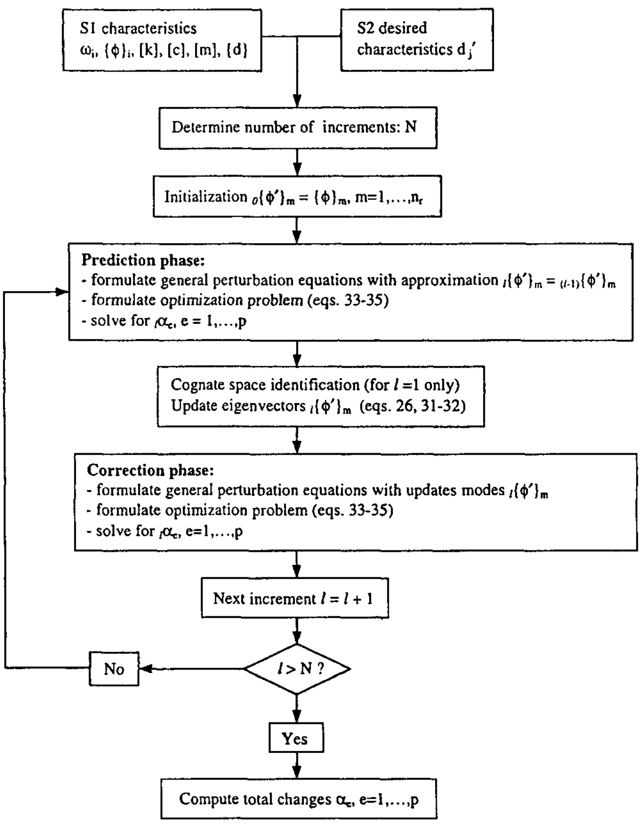

Fig. 2 Schematic representation of algorithm.

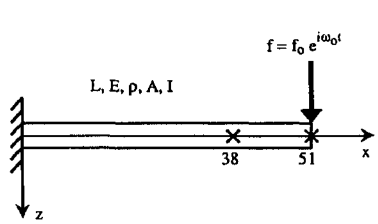

Fig. 3 Cantilever beam model: length, $L = 2.5$ m; modulus of elasticity, $E = 2.07\text{E}{+}11$ N/m^2; mass density, $\rho = 7200$ kg/m^2; cross-sectional area, $A = 5.0\text{E}{+}3$ mm^2; moment of inertia, $I = 1.042\text{E}{+}6$ mm^4; load amplitude, $f_0 = 9.81$ N; frequency of excitation, $\omega_0/2\pi = 100$ Hz; and damping coefficients, $a_1 = 0.14$, $a_2 = 0.01$.

III. Redesign Applications

A. Redesign of a Proportionally Damped Cantilever Beam

A simple cantilever beam excited at its free end at 100 Hz is shown in Fig. 3. The finite element model contains 50 elements, 4 element sets, and 2 redesign variables per element set (cross-sectional area and moment of inertia); see Fig. 4.

The results of several redesigns are presented in Table 1. The tabulated information can be interpreted as follows. In case 1 only the first nine eigenvectors are used ($n_r = 9$). The total change is divided into 7% increments (IC = 0.07). One constraint is applied: the forced response amplitude at node 51 is required to be 1.1 times larger (redesign goal: $d_{51} = 1.1$). After redesign a FEA of the obtained structure is performed to give the actual response. The reader must see that this final FEA is only performed to check the accuracy of the results and is not part of the solution process. In case 1 of Table 1, the actual response is found to be 1.101 times larger than the response of the initial structure. The corresponding relative error of 0.09% is |1.101—1.1|/1.1. The following observations can be made based on Table 1.

B. Discussion

The first three cases in Table 1 show that accuracy is reduced with increasing redesign changes. It still remains high for changes by a factor of 2.0, which is achieved without a single finite element analysis. Cases 3 and 4 and 10–12 show that accuracy can be improved by decreasing the percentage of incremental changes. Smaller increments lead to a proportionally longer running time with less error generated by the approximation of the extracted modes in the prediction phase. Accumulation of numerical error, however, is expected to be greater, which still does not have a significant effect in these cases. The number of extracted modes is an important issue as far as accuracy and running time are concerned. Theoretically, an infinite number of modes would yield the exact solution of the problem. Practically, there exists a minimum number of modes required to meet a satisfactory accuracy for each case. On the other hand, higher accuracy results in computational time, which is directly proportional to the number of modes used. There are three major computational tasks in the LEAP algorithm: the development of the perturbation equations in the prediction phase, the computation of the admixture coefficients, and the development of the perturbation equations in the correction phase. Taking into perspective the magnitude of the these computational tasks, the resolution of the optimization problem does not take as much time as the development of the problem itself. Cases 6 and 7 illustrate that accuracy increases with the number of modes. In cases 8, 9, and 11 two constraints are defined in the redesign. In case 8 the structure becomes stiffer, and in cases 9–11 more flexible. Results are accurate even for very large changes such as in case 11. In cases 13–17 the achieved changes are not as large as those achieved in case 11. They are considered, however, more difficult cases by the fact that the two constraints correspond to conflicting requirements. In cases 13 and 14 the new design must be stiffer to reduce the amplitude of vibration at node 38 while increasing the amplitude at the free end node 51. No solution to this redesign problem was found even with a high number of eigenvectors and small increments (case 13). The alternative objective function presented in Sec. II.B was used (cases 14) to obtain a solution with satisfying accuracy. A penalty term minimizing the difference between the moments of inertia of adjacent element sets and the difference between cross-sectional areas of adjacent element sets was introduced in the objective function. The result was a smooth distribution of moment of inertia and cross-sectional area along the beam with high accuracy. In cases

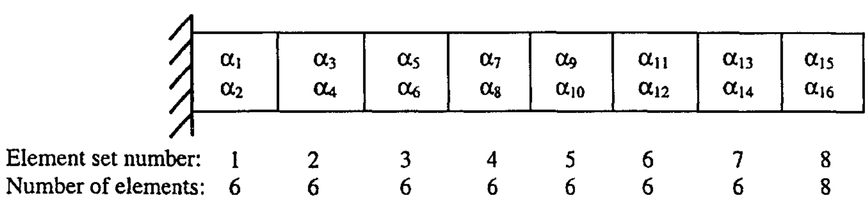

Fig. 4 Element sets and redesign variables. Number of finite elements = 50, number of degrees of freedom = 100, number of element sets = 8, number of redesign variables per element set = 2 (cross-sectional area, moment of inertia), and total number of redesign variables = 16.

Table 1 Redesign of cantilever beam

Case	n_r[a]	IC[b]	Redesign goals[c]			Obtained response[d]			Error, %[e]			CPU, s
			d_{38}	d_{51}	ω_1^2	d_{38}	d_{51}	ω_1^2	d_{38}	d_{51}	ω_1^2	
1	9	0.07	—	1.1	—	—	1.101	—	—	0.09	—	25
2	9	0.07	—	1.5	—	—	1.520	—	—	1.35	—	70
3	9	0.07	—	2.0	—	—	2.045	—	—	2.23	—	121
4	9	0.035	—	2.0	—	—	2.023	—	—	1.15	—	261
5	9	0.07	1.5	—	—	1.501	—	—	0.08	—	—	114
6	3	0.07	2.0	—	—	2.009	—	—	0.44	—	—	23
7	9	0.07	2.0	—	—	2.005	—	—	0.25	—	—	58
8	9	0.07	0.5	0.5	—	0.500	0.512	—	0.15	1.93	—	112
9	9	0.07	1.5	1.5	—	1.503	1.516	—	0.25	1.09	—	42
10	9	0.035	2.0	2.0	—	2.003	2.014	—	0.17	0.78	—	239
11	9	0.07	2.0	2.0	—	2.005	2.026	—	0.33	1.55	—	126
12	9	0.15	2.0	2.0	—	2.015	2.059	—	0.76	3.24	—	54
13	18	0.035	0.75	1.5	—		No solution found		—	—	—	690
14[f]	9	0.07	0.75	1.5	—	0.755	1.526	—	0.62	1.75	—	442
15	9	0.07	1.3	0.85	—	1.329	0.862	—	2.18	1.46	—	36
16	9	0.07	1.5	0.75	—		No solution found		—	—	—	44
17[f]	9	0.07	1.5	0.75	—	1.399	0.761	—	6.47	1.55	—	101
18	18	0.07	1.5	1.5	1.5	1.502	1.514	1.500	0.18	0.96	0.03	99
19	18	0.07	1.5	1.5	1.0	1.506	1.518	1.002	0.43	1.23	0.22	99
20	18	0.07	1.5	1.5	0.75	1.506	1.520	0.708	0.41	1.34	5.56	100

[a] n_r: number of extracted modes in modal expansion.
[b] IC: Incremental change, e.g., 0.07 corresponds to 7% change increments.
[c] Redesign goals: ratio between goal and initial response.
[d] Obtained response: ratio between obtained response after redesign and initial response.
[e] Error, %: 100 × |obtained response − redesign goal|/redesign goal.
[f] A penalty term was used in objective function (see Sec. II.B).
Each run was executed on HP workstation C-160 (processor: 160 MHz PA-RISC 8000).

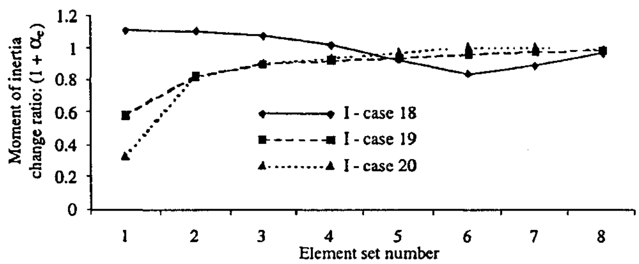

Fig. 5 Change in moment of inertia: cases 18–20.

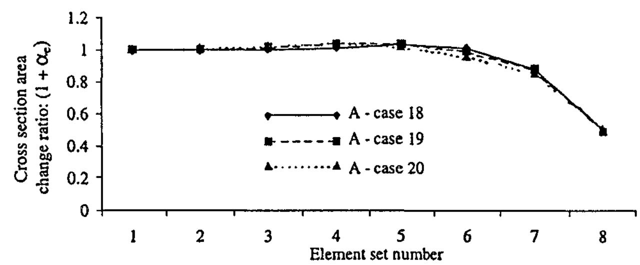

Fig. 6 Change in cross-sectional area: cases 18–20.

15–17 conflicting requirements are imposed. The amplitude at node 38 is required to increase and the amplitude at the free end to decrease. In case 16 the amplitude at node 38 is still twice as small as the one at node 51. No feasible solution was found because of the uneven distribution of the moment of inertia and cross-sectional area along the beam. In case 17 the same penalty term as in case 14 is introduced. A solution is found with a rather large error because requirements are greatly conflicting. In the final three cases, a third constraint is added to the problem, which is a redesign requirement on the first natural frequency. The two constraints on nodes 38 and 51 correspond to compatible requirements. These two constraints alone (case 9) result in a higher first natural frequency for the new design, mainly because of the reduction of the cross-sectional area around the free end of the beam, which corresponds to a decrease in mass where the motions are the greatest. Therefore, by constraining the first natural frequency and requiring it to be larger (case 18), great accuracy is expected. By requiring it to remain the same or decrease (cases 19 and 20 respectively), accuracy is decreased because these constraints correspond to conflicting requirements. Still, a solution is found with relatively good accuracy. From cases 18–20 one can notice that the computational time does not increase with the difficulty of the problem when a solution exists. Only the number of increments and the number of extracted modes affect the computational time. The results of cases 18–20 are graphically represented in Figs. 5 and 6. Figure 6 shows that the mass distribution, which is directly related to the cross-sectional area distribution, does not have much effect on the change in the first natural frequency. For the three cases the mass is decreased at the free end of the beam, whereas the stiffness, which is directly related to the moment of inertia, is changed mainly at the clamped end.

C. Redesign of a Two-Dimensional Cross Section of a Ship Model

A simplified cross section of a model of a ship, shown in Fig. 7, is excited on one side by a point force at a frequency of 100 Hz. The rigid body modes are eliminated by introducing single point constraints at the bottom and the top of the structure. The structure is discretized with 79 beam finite elements.

D. Discussion

The number of finite elements, 79, was chosen in such a way that, for the 18th eigenmode of the structure, each vibrational wave contains at least 6 finite elements. More elements would increase the computational time without improving accuracy. The results of several redesigns are presented in Table 2. They confirm the conclusions drawn from the beam redesign problem. Additional interesting points are discussed.

Cases 1–4 show that accuracy increases with increasing number of extracted modes. The computational time increases proportionally while the error (1.82%) at node 14 remains satisfactory. Accuracy is improved in case 5 where the eigenvectors are updated at the end of each correction phase in addition to being updated at the end of each prediction phase. The discussion on the choice of the objective function in Sec. II.B is illustrated with the respective comparison of cases 6, 10, and 18 to cases 5, 9, and 17. It shows that accuracy is improved when a penalty term is used in the objective function to produce smoothly distributed changes along the structure [see Eq. (36)]. In cases 5 and 9 the eigenvectors are updated at the end of each correction phase. Compared respectively to cases 4 and 8, the accuracy is improved but the computational time is quite large. Cases 7 and 8 correspond to smaller increments. The accuracy is slightly improved as compared to cases 2 and 4. In cases

Table 2 Redesign of ship model cross section

Case	n_r[a]	IC[b]	Redesign goals[c] d_{14}	d_{28}	ω_1^2	Obtained response[d] d_{14}	d_{28}	ω_1^2	Error, %[e] d_{14}	d_{28}	ω_1^2	CPU, s
1	3	0.07	1.5	1.5	——	No solution found			——	——	——	24
2	5	0.07	1.5	1.5	——	1.509	1.323	——	0.56	11.8	——	34
3	9	0.07	1.5	1.5	——	1.525	1.522	——	1.67	1.52	——	65
4	18	0.07	1.5	1.5	——	1.528	1.507	——	1.82	0.47	——	154
5[f]	18	0.07	1.5	1.5	——	1.512	1.508	——	0.78	0.58	——	189
6[h]	18	0.07	1.5	1.5	——	1.499	1.497	——	0.07	0.19	——	403
7	5	0.035	1.5	1.5	——	1.506	1.320	——	0.38	11.9	——	136
8	18	0.035	1.5	1.5	——	1.525	1.506	——	1.64	0.45	——	545
9[f]	18	0.035	1.5	1.5	——	1.510	1.500	——	0.65	0.00	——	392
10[h]	18	0.035	1.5	1.5	——	1.499	1.496	——	0.10	0.24	——	859
11	18	0.07	0.75	0.75	——	0.753	0.753	——	0.40	0.42	——	111
12	18	0.07	0.5	0.5	——	0.505	0.516	——	1.01	3.15	——	214
13	18	0.035	0.5	0.5	——	0.502	0.515	——	0.57	3.00	——	415
14	18	0.07	0.75	1.5	——	0.774	1.348	——	3.20	10.4	——	132
15	18	0.07	1.5	0.75	——	1.576	0.372	——	5.00	49.40	——	302
16	18	0.07	1.5	1.5	0.75	1.532	1.488	0.730	2.10	0.79	1.38	348
17	18	0.07	1.5	1.5	0.5	1.549	1.436	0.457	3.24	4.25	4.13	766
18[g]	18	0.07	1.5	1.5	0.5	1.499	1.498	0.500	0.06	0.11	0.02	374
19	18	0.07	1.5	1.5	1.5	1.514	1.503	1.491	0.95	0.20	0.31	149
20	18	0.07	0.75	0.75	1.5	0.753	0.750	1.508	0.45	0.04	0.20	132
21	18	0.07	0.75	0.75	0.75	0.753	0.751	0.749	0.40	0.20	0.08	162

[a]n_r: number of extracted modes in modal expansion.
[b]IC: Incremental change, e.g., 0.07 corresponds to 7% change increments.
[c]Redesign goals: ratio between goal and initial response.
[d]Obtained response: ratio between obtained response after redesign and initial response.
[e]Error, %: 100 × |obtained response − redesign goal|/redesign goal.
[f]Eigenvectors were also updated at the end of correction phase.
[g]Penalty term in objective function.
[h]Eigenvectors updated in correction phase and penalty term used in objective function.
Each run was executed on HP workstation C-160 (processor: 160 MHz PA-RISC 8000).

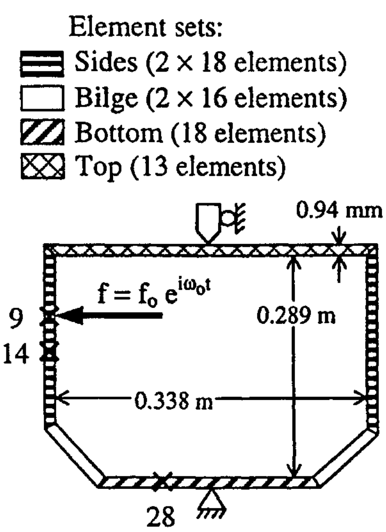

Fig. 7 Idealized cross section of a ship model with beam elements: modulus of elasticity, $E = 2.07$ E+11 N/m^2; mass density, $\rho = 7200$ kg/m^2; cross-sectional area, $A = 23.9$ mm^2; moment of inertia, $I = 1.76$ mm^4; load amplitude, $f_0 = 44.4$ N; frequency of excitation, $\omega_0/2\pi = 100$ Hz; damping coefficients, $a_1 = 0.10, a_2 = 0.01$; number of finite elements, 79; number of degrees of freedom, 234; number of element sets, 4; two design variables per element set (A, I); excitation at node 9; constraint at node 14, horizontal motion, and constraint at node 28, vertical motion.

11–13 the structure is required to become more rigid or heavier in order to reduce the vibration amplitudes. In cases 14 and 15 conflicting requirements are imposed. The results show a significant error, which is caused by the fact that the required design is unfeasible and the best compromise solution with violated constraints is found. By using a higher number of redesign variables and a more adequate objective function, a solution (expected to exist) could probably be found. In the final five cases a constraint on the first natural frequency is imposed. The results are very accurate in cases where the vibration amplitudes are reduced (cases 20 and 21). Cases 4 and 19 show that with the additional constraint on the first natural frequency the results are more accurate than without, which is explained by the fact that in the algorithm the eigenvectors are updated at each increment at the end of the prediction phase using the predicted solution. The eigenvectors are not updated a second time at the end of the correction phase using the corrected solution. Therefore, a small error is expected to build up with the number of increments, and constraining the first natural frequency has a similar effect as updating the first eigenmode at the end of the correction phase. Even though the constraints correspond to conflicting requirements, good accuracy is achieved. Obviously, in cases 17 and 18, because the objective functions are slightly different, the solutions found are also different.

IV. Conclusion

A large admissible perturbations methodology for structural redesign for forced response amplitude of undamped and proportionally damped vibrations was developed. The corresponding LEAP algorithm was implemented in code RESTRUCT. Sources of damping were modeled by proportional damping, which allows the derivation of exact perturbation equations. The new algorithm was tested for a cantilever beam with 100 degrees of freedom and a simplified cross section of a ship model with 234 degrees of freedom. Extremely good accuracy was obtained for small changes, and satisfactory accuracy for large changes. For conflicting requirements, the choice of objective function appears to be a significant factor in finding an accurate solution, which introduces undesirable subjectivity into the redesign process. The LEAP redesign algorithm was shown to require only one finite element analysis and to produce satisfactory results for large changes.

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A. D. Belegundu
Associate Editor